**N-body Simulations of Star-Star Encounters**

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**Abstract**

N-body simulations were carried out on a self-gravitating system, like the Solar System. A summary of all the sections is mentioned in the introduction. The N-body problem is described with all the equations used in the code. The next section explains the motivation of writing this N-body code from its application to the astrophysics problems. A literature review explaining the N-body problem and its application to problems in astrophysics is included. The progress section includes the description of the second-order code applied and the results produced for the solar system.

**1. Introduction**

**2. Literature review**

**2.1 N-body problem**

Newton’s law of gravitation describes how a group of stars interact in a star cluster. These gravitational interactions cause the dynamical properties (velocity, position, acceleration) of the stars to change. This causes a change in the dynamical properties of the whole region. The evolution of this stellar region can be observed in a N-BODY simulation. The N-BODY problem is incorporates the prediction of future dynamical properties of the stars in the system. The initial properties of the particles in the system are known. In a system of N particles, the acceleration of a particle can be defined as,

-1

Where, mass of the test particle

the unit vector along the direction of the distance vector

the modulus square of the distance between the bodies considered

Integrating equation (1) provides the position and velocity of a particle at any time t. For *N*=2 the above equation is analytically solvable. Since we are considering stellar clusters as our system, where N2, numerical methods are considered.

Numerical integration of equation 1 provides the below solutions.

-2

-3

Where, and the new position and velocity of the particles.

and the initial position and velocity of the particles

, , , the initial acceleration of the particles with the latter three being the 1st, 2nd, 3rd time derivatives.

timestep for the simulation

The timestep *dt* determines the accuracy of the values of the future position and velocity of the particles. It goes inversely with the computational time. There is a rise in the number of calculations done in a single simulation as *dt* drops.

Accuracy in predicting the motion of stars in the cluster is the main task. The error in the solution is proportional to the timestep. As we go to higher orders, a small drop in *dt* will imply a large reduction in the error.

For our project we consider the 2nd order method, with the 4th order predictor-corrector (Hermite scheme) method used in semester 2. Below are the equations used for the second order method.

-4

-5

The 4th order predictor-corrector method improves on the accuracy of the previous methods.

**2.2 History of N-BODY simulation**

**2.2 Astrophysics applications of N-BODY method**

The motivation of writing this N-body code is due to its application to certain astrophysics problems. The problems are: mass segregation in star clusters, the orbital migration of the giant planets in the planetesimal disk, presence of boxy and peanut-shaped bars in the bar evolution phase. In these problems, we have the presence of self-gravitating system. Newtonian gravity is the dominant factor in the system.

Star cluster evolution is a difficult scenario to build in a numerical simulation. The interaction between the stars determine the evolution of the cluster. In Khalisi, E., Amaro-Seoane, P. and Spurzem, R. (2006) the dynamical evolution of the star cluster was carried out by considering two different stellar mass groups. In dynamical equilibrium, the evolution of the cluster depends on it achieving a thermal velocity distribution through small changes in the velocity of the stars. This phenomenon is termed as relaxation. Relaxation forms the major part in shaping the structure of the cluster. The important phase in the evolution of the cluster comes during the central core collapse. Three processes are able to reach this stage: equipartition, evaporation and gravothermal instability. Star-star encounters allow mass segregation in the central regions. This leads to movement in the heavy and low mass components to and away from the centre of the star cluster. This movement is achieved through transfer of Kinetic energy from the heavy to light component. There is a thermal energy outflow from the center to the outer regions. Significant evidence for mass segregation in star clusters was found in Infra-red observations of the trapezium cluster in Orion.

A simplest approximation of two-mass component simulations is considered. To carry out the simulations, they considered a plummer sphere model of the density distribution in the cluster. with the particles considered point-like, close encounters are not avoided. In addition, other stellar evolution scenarios, like primordial binaries, tidal fields etc. They use two parameters, the fraction of heavy mass component, and the mass fraction of each particle, to describe each model. A number of runs were assigned to each model, with each of them having a different setup of initial positions and velocities for all the particles. A strong approach was done to produce accurate N-body simulations with a higher number of particles, keeping in mind about a low computing power. With mass segregation in stellar clusters used for N-body simulations, the orbital migration of the giant planets in a planetesimal disk also forms a N-body problem.

The Nice model explains the orbital migration of the giant planets and the current structure of the outer Solar System. Specifically the orbital migration of Neptune caused the for majority of KBOs at Neptune’s mean motion resonance of 5:2. Also the large eccentricity in the Pluto’s orbit came from this migration. The Kuiper belt inhabits a vast number of residual planetesimals which are at various resonances. These Kuiper Belt Objects form a relic of the history of the outer solar system. The occurrence of the KBOs came from the orbital migration of the giant planets. The interaction between the planetesimal disk and the planets led to exchange of angular momentum thereby affecting the orbits.

N-body simulations of the interactions between the giant planets and the planetesimal disk was carried out (Hahn, J.M. and Malhotra, R., 2005, p. 2392). The simulations included massless particles surrounding the migrating giant planets. With a timescale of the age of the solar system, the orbital evolution was tracked. Two scenarios for the initial disk were considered: when it was dynamically cold, and the final where it was destabilized before migration. A MERCURY6 N-body integrator for the particular evolution process. The migration was demonstrated by applying an external torque to the semi-major axis of the orbit. Below is the time-dependent form of the semi-major axis,

Where, the final semi-major axis of the planet

planet’s net radial displacement

e-fold timescale for planet migration

For the initial conditions, the current masses and orbits of the planets was considered. In addition to that, the initial semi-major axis was modified by applying . This allowed for the torque to return the orbits to their present configurations. Also, various constraints were used for for each planet’s orbit depending on the different resonances. A timestep of 0.5 years was used, with the massless particles randomly distributed over a range of 20-80 AU. A more extensive approach for the N-body simulations was done by Hahn, J.M. and Malhotra, R. (1999, p. 3041) to look at the orbital migration of the giant planets in the remnant planetesimal disk. Moreover, different cases for the migration were considered based on the mass of the planetesimal disk.

Another case which can be explained using N-body simulations is stage after the runaway growth. The core accretion theory for planet formation gives a good description of the runaway growth and the oligarchic stages. With the protoplanets created, their orbital evolution among the remaining planetesimals was explored using 3D N-body simulations by Kokubo, E. and Ida, S. (1998, p.171). They observed the growth of the protoplanets slower among them, whereas when compared to the growth of the planetesimals. A formation of planet-planetesimal system was explored.

The formation of the magellanic stream near the Milky Way through galactic cannibalism is produced using numerical simulations (Maddison, S.T., Kawata, D. and Gibson, B.K., 2002, pp.421-422). The tidal gravitational field formed from the merger of the MW and the Large and Small Magellanic Clouds were the cause for this stream. They compared two types of numerical simulations, them being the N-body only merger and hydrodynamic one. For the latter type, the factors of star formation, supernova feedback and metal enrichment were included into the simulations along with the gravitational interactions. The results from the N-body only simulations showed the SMC being tidally stripped which gave the presence of the magellanic stream. However, from the second simulation the SMC was strongly disrupted with the stream devoid of stars. Figure 1 explains these results. The observations of the magellanic stream depict most of the gas in the stream.

A close up of a map

Description automatically generated

Figure 1: the different panels show the results from the N-body only and hydrodynamical simulations of the three-galaxy merger. Left two panels- N-body-only results showing the magellanic stream. Three right panels- hydrodynamical simulations results show the presence of gas in the magellanic stream.

**3. Progress on project**

The initial work on the project was carried by constructing a simple second order code. Equation 5 can be corrected by considering an assumption.

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**3.1 Three-body system**

Initially we a three-body system of SUN-EARTH-JUPITER was considered for problem.

For the first block of the code, the declaration of all the variables used was done. For certain known variables like the gravitational constant *G*, the mass, velocity and initial positions of the planets, the initialisation is done. These values were obtained from the planetary fact sheet in the NASA website[1]. Initial conditions for the position and velocity were given. The orbits of the planets were forced to be on the XY plane with the initial velocity equal to the y component.

For this system, we added the code which did centre-of-mass and velocity corrections. Since there is a common centre-of-mass for the Sun-Earth-Jupiter, we stop the system from drifting away from the origin during the simulations. The centre-of-mass and centre-of-velocity can be calculated as below,

To check the stability of this system, an energy conservation check had to be done. this required the initial energy to be determined using the values initialised above.

To carry out the simulations, an infinite loop was constructed which would up-to a simulation time (which is assigned before the loop). Along with this, a value for the timestep *dt* was also selected. Using the initial position of the objects, the future positions were first determined using equation 4. Then an acceleration loop was created which calculated the future accelerations of the bodies using equation 1.

We declare all the variables (, etc.) to be included in the code. The initialisation of the known variables is done (i.e. setting , number of bodies, *n*=3, etc.)

For this system, we set certain initial conditions for the velocity and position of the bodies.

This forces the bodies to orbit in the XY plane, starting from the X axis with the initial velocity in the Y direction.

The most important part of this code comes in creating an acceleration loop. This loop is used in determination of the initial acceleration, and the future acceleration, .

We produce a time loop which runs up-to 30 yrs. With the initial and future accelerations in hand, the future position and velocity of the bodies is calculated using equation 4 and 6. A figure is produced of the orbits of the three bodies using the x and y positions. The orbits of the Sun-Jupiter-Earth will drift away from the origin. However, it is not seen here since the running time was significantly short. This is due to the presence of a common centre of mass for the bodies which was not included at the beginning.

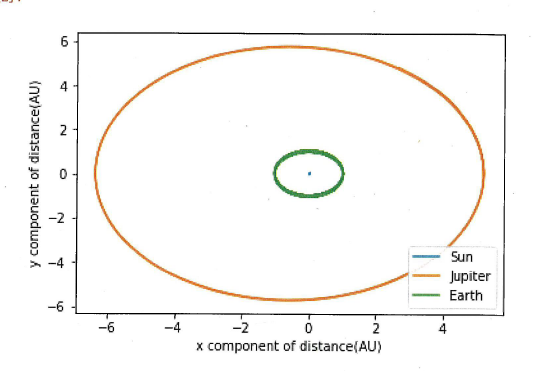


Figure 1: Orbits of the Sun-Jupiter-Earth with the simulations run for 30 yrs. The orbits are not corrected for the centre-of-mass and velocity.

To correct that, we introduce the centre-of-mass and velocity corrections (given in the code in the appendix). Then with the runtime changed to 100 yrs, the orbits did not show the drifts.

The stability of this system is checked using the fractional energy calculation using the equation given below.

Where and – the initial and the current energy of the system (at the current time, )

This energy conservation check was done with the timestep at 100 sec.

**3.2 Whole solar system**

To make the code more useful, the whole solar system is included. The known values of the position, velocity and mass are given. These values are taken from the Planetary fact sheet in the NASA website[1]. The simulations are made to run to 1000 yrs.

For this system, the energy conservation check is carried out, with a timestep of 1000 seconds. For every 6 months, the fractional energy values are noted. A fractional energy plot is produced for this system.

A screenshot of a cell phone

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Figure 2: Fractional energy varying with time (in yrs) for the planets in the solar system.

Due to the very small values of the fractional energies, the system is stable.

Another stability test comes from investigating the how the distance from the sun () varies for the planets. The simulations are run for 1000 yrs with a similar timestep used. To reduce the computation time, the values for are taken at every 1000th iteration.

A screenshot of a social media post

Description automatically generatedFigure 3: Distance from the Sun (AU) against time (yrs).

The periodically varying curves give good idea of the stability of this system. The features in the curve for Saturn arrive from its interactions with Jupiter.

4. Project plan

At the start of week 1, the core part of the work in semester 2 begins with task1, i.e., building a basic fourth order predictor-corrector code. A time of 2-3 weeks is given for this task based on its difficulty. After forming the base, we require an adaptive timestep for the code (task 2). Depending on the errors obtained from the energy checks, the code either doubles or halves the timestep, *dt*. A similar amount of time of 3 weeks is assigned to this task. The two main components of the code are ready. This leads to task 3 of testing the code. The results obtained from these tests determine whether the code is working. For example, we should observe milankovic cycles when we produce the plot of the orbits of the planets in our Solar System.

Task 4 provides the motivation of writing this piece of code. We apply this code to any astrophysics problem (like late oligarchic phase of planet formation). Moreover, certain tweaks are added at this point to increase the speed of the code. With a time of 2 weeks, it overlaps with the easter break. Then we lead to the most important task 5, the write-up of the report. All the figures and results produced in tasks 3 and 4 are included in the report. Tasks 4 and 5 overlap at the beginning of easter break. A time of 3-4 weeks is assigned so that a draft of the final report can be submitted to the supervisor approximately two weeks before the deadline (17/05/2019). The final task involves in refining the report. Within a week the report is checked for any mistakes before the final submission.

**5. Conclusion**

**References**

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<https://nssdc.gsfc.nasa.gov/planetary/factsheet/>

* Neptune migration into the destabilised the Kuiper belt- the interaction of Neptune with the destabilised Kuiper belt causing the formation of Kuiper belt objects
  + PROBLEM TO ADDRESS-to show the presence of the current Kuiper belt objects at Neptune’s 5:2 resonance due to the outward migration of Neptune into a stirred Kuiiper belt.
* Kuiper belt- residing beyond 30 AU, contains a vast number of residual plantesimals from the primordial planetesimal disk. This debris was not able to merge to form planetary mass objects.
* This belt, the relic of the outer solar system’s history.
* The orbital migration of the giant planets came from its interaction with the planetisimal disk, leading to exchange of angular momentum.
* This planet-migration scenario was the cause for pluto to gain a peculiar orbit, it being captured at neptune’s 3:2 resonance. This migration caused large eccentricity in pluto’s orbit. Further evidence of this scenario came from the discovery of KBOs at 3:2 resonance.
* This is a planet-migration model, explaining why neptune’s outward migration caused KBOs at 5:2 resonance.
* Carry out N-body simulations of the interaction between the migrating giant planets and massless particles (representing the KBOs). Track the orbital evolution of these planets where the timescale is the age of the solar system.
* They carry out two simulations: one where the the Kuiper belt is initially dynamically cold, and the second where the disk is initially disturbed before the migration.
* Used a MERCURY6 N-body integrator (Chambers 1999) used to track the orbital evolution of the four giant planets with the vast number of particles. Here, migration resulted from applying an external torque to the planet’s orbit which gave a time-varying semi-major axis.

Where, the final semi-major axis of the planet

planet’s net radial displacement

e-fold timescale for planet migration

* The simulations used the current planets’ masses and orbits as the initial conditions. But the initial semimajor axis is displaced by so that the external torque moves the planets to the current orbits. Various constraints on the values of for the planets were applied, keeping in mind the various resonances.
* For the simulations the timestep is chosen to be 0.5 years. The massless particles with their semi-major axes randomly distributed over a range of 20-80 AU.
* NICE model- describes the outward migration of the outer planets, the destabilisation of the Kuiper belt and the current scenario of the Neptune and Uranus.
* Mass segregation in star clusters
  + PROBLEM ADDRESSED- internal evolution of a star cluster which is in dynamical equilibrium. With 2 body encounters between the stars, the cluster achieves a thermal velocity distribution which dictates this evolution. This process is termed as relaxation.
  + Central core collapse is important in this cluster evolution. This collapse is achieved by different processes happening: equipartition, evaporation and gravothermal instability.
  + We observe mass segregation in the central regions of the cluster through these star-star collisions.
  + The heavy component drifts to the centre by transferring KE to the lighter component. This leads to equipartition condition for the cluster to achieve dynamical and thermal equilibrium. However most clusters are self gravitating systems, where equipartition is not effective. As thermal energy outflow occurs from the inner to outer region, the core loses heat contracts. This doesn’t affect the outer regions.
  + Significant evidence for mass segregation was found in clusters observed. McCaughrean and Stauffer (1994) and Hillenbrand and Hartmann (1998) done IR observations of the trapezium cluster in orion. Clearly showed that mass segregation occurs with the high mass stars occupying the central regions.
  + To model this evolution, they considered a simple bi-modal mass spectrum as the start (heavy component and the light component(s)). These are two-mass simulations.
  + Used a plummer sphere model (a model for the density distribution of the star cluster) which is in global virial equilibrium. The particles are considered point-like, adding the fact that close encounters are present.
  + Two different mass species were considered as the simplest approximation.
  + They ignored other stellar evolution processes, presence of primordial binaries, cluster rotation and tidal field. Just aimed in isolating essential mass segregation processes.
  + Here, each model determined by two parameters; fraction of the heavy mass component, and the mass fraction of individual particles, , where the mass of the particles in the heavy component and the light component.
  + Each model is assigned a number of runs. Each run differs in the random number seed which gives a setup of initial positions and velocities to the particles. The distribution function is the same for each run.
  + They wanted to produce accurate N-body simulations for certain situations of the star cluster like thermal and dynamical equilibrium. This star cluster is considered as a self-gravitating system (which we also define in our simulations). Newtonian gravity is the important factor. This work was done by von Hoemer in 1960. Performed N-body simulations with number of particles/stars = 16. Limited computing power caused a restriction here. With development in the hardware side and the software, more accurate calculation were done with large number of particles/stars.
* NICE model explaining the origin of the Kuiper belt and its evolution
* Orbital evolution of giant planets while embedded in the planetesimal disk-
  + THE PROBLEM ADDRESSED-
    - Formation of the Kuiper belt and the oort cloud after the gravitational clearing of the remnant planetesimal disc.
    - Interactions between the giant planets and the planetesimals caused exchange of angular momentum and energy. This led to orbital expansions of the planets like Uranus, Neptune and Saturn.
    - These planetesimals start getting captured at various mean motion resonances of Neptune. The result of this causes a non-uniform orbital distribution of the Kuiper Belt Objects with varying eccentricities and inclinations for their orbits.
    - Therefore, look at the planet-migration scenario where we do numerical simulations of the evolution of the giant planets in the remnant planetesimal disc. Assumed the disc to contain low-mass particles.
    - The numerical method-
      1. Model involved the sun and the four giant planets with a population of low mass particles distributed in a disc.
      2. The ideal number of particles was . But there was restrictions in the computing power at that time.
      3. They used a fast body integrator with simplifications.
      4. Assumed each body as a point mass particle. It is justified since in the late stages of planet formation, the likeliness of planet-particle collisions is minimal. Since gravitational interactions dominate at that point.
      5. Another assumption- included the mutual gravitational forces between the planets and the forces between the planet and low mass particles. Neglected the mutual interactions the particles to avoid huge computational expense.
      6. A restriction placed on the upper limit of the semi-major axes of the particles. Particles where *a*>3000 AU would be removed. In dynamical models, particles scattered in wide orbits > 5000 AU get decoupled from the planets due to a galactic tide. These merge into the oort cloud.
      7. Used a second order mixed variable symplectic (MVS) mapping- this allowed for rapid advancement of the heliocentric positions and velocities of the planets and low-mass particles as they interact in the Sun’ Gravitational field.
  + A presence of residual planetesimal disk with mass 10-100 solar masses in the vicinity of the giant planets.
  + Planetary migration causing the disk to disappear due to the exchange of angular momentum between the planets and the planetesimals in the disk.
  + Final stage of planet formation- oligarchic phase. We see the clearing of the residual planetesimal disc by giant planets. When the giant planets formed, there would have been a residual planetesimal disc. But due to the gravitational interactions, this disc was cleared out. This affected the orbits of the planets as there was exchange of angular momentum involved.
  + Formation of the oort cloud coming from this stage. Mass of this cloud 10-100 solar masses, at distances of 1000-100000 AU from the sun.
  + Outward migration of Neptune- this migration of Neptune in the disturbed Kuiper belt caused several KBOs to be captured at various mean motion resonances of Neptune. Mainly we see a significant number of KBOs at a 3:2 resonance with an eccentricity of 0.1-0.35. we see a non-uniform distribution of KBOs orbits. Also pluto was captured at one of the resonances along with other KBOs.
  + There would be orbital expansion due to the gravitational clearing by Neptune. The semi-major axes and eccentricities of the KBOs should rise along with this expansion. From the analysis of the resonance-sweeping mechanism, if the planet migration of Neptune caused the eccentric and inclined orbits of pluto and KBOs, this would expand the orbit of Neptune by 5-10 AU.
  + There is a non-uniform KBO orbital distribution. This can provide the orbital migration history of Neptune.
* Oligarchic growth of protoplanets
  + Investigating the orbital evolution of protoplanets surrounded by large number of planetesimals using 3D N-body simulations.
* Bar diagnostics in edge-on spiral galaxies- N-body simulations of disks